

$$\begin{aligned}
 \text{Ti) 5) b)} \quad [A|b] &= \begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & \lambda & 5 \\ 1 & 2 & 4 & k \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & \lambda-2 & 2 \\ 0 & 1 & 2 & k-3 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & \lambda-2 & 2 \\ 0 & 0 & 4-\lambda & k-5 \end{bmatrix}
 \end{aligned}$$

i) if $\lambda-4=0$, $k-5 \neq 0$ no sol.

ii) $\lambda-4=0$ & $k-5=0$ inf. many $\lambda=4, k=5$

iii) $\lambda \neq 4$ unique sol.

$$(4-\lambda)z = k-5 \Rightarrow z = \frac{k-5}{4-\lambda}$$

$$y + (\lambda-2)z = 2 \Rightarrow y = 2 - \frac{(\lambda-2)(k-5)}{4-\lambda}$$

inf. many

$$\begin{aligned}
 &\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
 &\begin{matrix} x & y & z \end{matrix} \\
 &= \begin{bmatrix} \textcircled{1} & 0 & 0 & 1 \\ 0 & \textcircled{1} & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}
 \end{aligned}$$

$$z = k_1$$

$x, y, z \rightarrow L.V.$

$z \rightarrow F.V.$

$$y + 2z = 2$$

$$y = 2 - 2k_1$$

$$x = 1$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2-2k_1 \\ k_1 \end{bmatrix}$$

$$B = \left\{ (x_1, x_2, x_3) : x_1 + 3x_2 = x_3 \right\}$$

$$x \in B \quad (x_1, x_2, x_3) : x_1 + 3x_2 = x_3$$

$$y \in B \quad (y_1, y_2, y_3) : y_1 + 3y_2 = y_3$$

$$X = \alpha \quad \dots \quad \dots$$

$$Y \in B \quad (y_1, y_2, y_3) : y_1 + 3y_2 = y_3$$

$$\begin{aligned} \alpha X + \beta Y &= \alpha (x_1, x_2, x_3) + \beta (y_1 + y_2, y_3) \\ &= (\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2, \alpha x_3 + \beta y_3) \end{aligned}$$

$$T.P. \quad \alpha x_1 + \beta y_1 + 3(\alpha x_2 + \beta y_2) = \alpha x_3 + \beta y_3$$

$$LHS = \alpha x_1 + \beta y_1 + 3\alpha x_2 + 3\beta y_2$$

$$\alpha(x_1 + 3x_2) + \beta(y_1 + 3y_2) = \alpha x_3 + \beta y_3 = RHS$$

$$1) \quad U = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} ; a, b \in \mathbb{R} \right\}$$

$$X = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} \quad a_1, b_1 \in \mathbb{R}$$

$$Y = \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix} \quad a_2, b_2 \in \mathbb{R}$$

$$\alpha X + \beta Y = \alpha \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \beta \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha a_1 + \beta a_2 & 0 \\ 0 & \alpha b_1 + \beta b_2 \end{bmatrix} \in U$$

$$2) a \quad x_1 > 0, y_1 > 0 \quad \alpha x_1 + \beta y_1 \geq 0$$

$$\alpha = \beta = -1 \quad -x_1 - y_1 \leq 0$$

$$d) \quad x_1 x_2 = 0, y_1 y_2 = 0 \quad T.P. \quad \alpha x_1 + \beta y_1, \gamma(\alpha x_2 + \beta y_2) = 0$$

$$e) \quad x_2 \in \mathbb{Q}, y_2 \in \mathbb{Q} \quad T.P. \quad \alpha x_2 + \beta y_2 \in \mathbb{Q}$$

$$\pi \quad e$$